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If $a$, $b$, $c$, and $d$ are rational, and $a + b\sqrt{2} = c + d\sqrt{2}$, then $a = c$ and $b = d$.

# THEOREM FOR EQUALITY OF SURDS

Suppose $\sqrt{k}$ is irrational, and that $a$, $b$, $c$, and $d$ are rational.
If $a + b\sqrt{k} = c + d\sqrt{k}$ then $a = c$ and $b = d$.

We can easily show by counter-example that this theorem does not hold for rational radicals.

For example, $1 + 4\sqrt{4} = 3 + 3\sqrt{4}$ but $1 \neq 3$ and $4 \neq 3$.

**Example 16** **Self Tutor**

Solve for $x$ and $y$ given that they are rational:

**a** $x + y\sqrt{2} = 5 - 6\sqrt{2}$

**b** $(x + y\sqrt{2})(3 - \sqrt{2}) = -2\sqrt{2}$

**Example 16**

Self Tutor

Solve for $x$ and $y$ given that they are rational:

**a** $x+y\sqrt{2}=5-6\sqrt{2}$

**b** $(x+y\sqrt{2})(3-\sqrt{2})=-2\sqrt{2}$

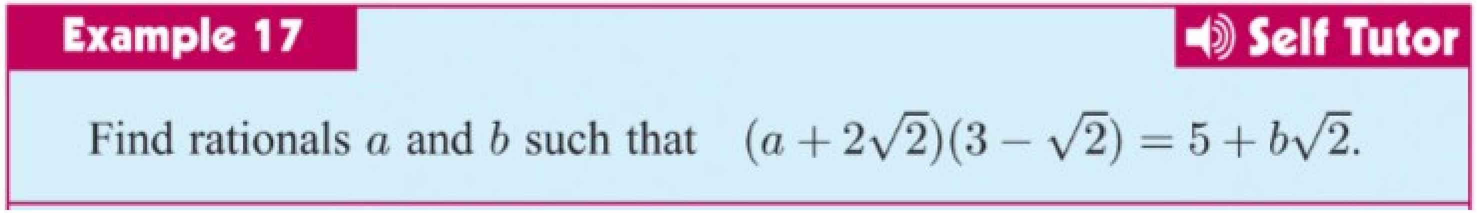
**a** Since $\sqrt{2}$ is irrational, $x=5$ and $y=-6$.

**b**
$$\begin{aligned}(x+y\sqrt{2})(3-\sqrt{2})&=-2\sqrt{2}\\ \therefore\ x+y\sqrt{2}&=\frac{-2\sqrt{2}}{3-\sqrt{2}}\\ &=\left(\frac{-2\sqrt{2}}{3-\sqrt{2}}\right)\left(\frac{3+\sqrt{2}}{3+\sqrt{2}}\right)\\ &=\frac{-6\sqrt{2}-4}{9-2}\\ &=\frac{-4-6\sqrt{2}}{7}\\ &=-\tfrac{4}{7}-\tfrac{6}{7}\sqrt{2}\end{aligned}$$

$\therefore$ since $\sqrt{2}$ is irrational, $x=-\frac{4}{7}$ and $y=-\frac{6}{7}$.

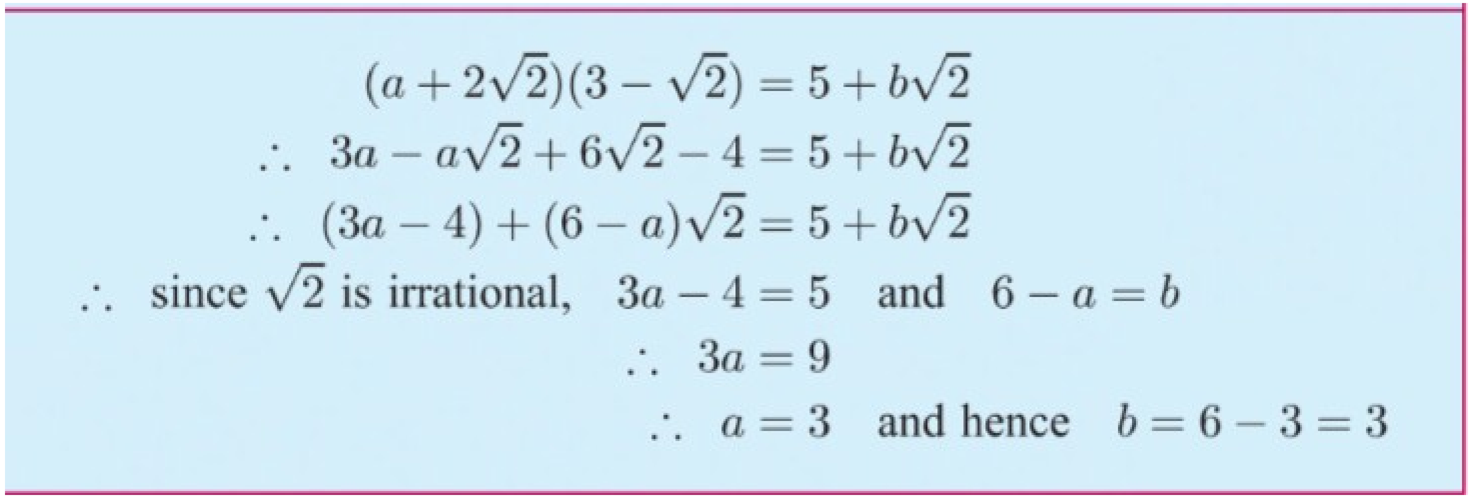
**Example 17** **Self Tutor**

Find rationals $a$ and $b$ such that $(a+2\sqrt{2})(3-\sqrt{2}) = 5 + b\sqrt{2}$.



**Example 17** **Self Tutor**

Find rationals $a$ and $b$ such that $(a+2\sqrt{2})(3-\sqrt{2})=5+b\sqrt{2}$.

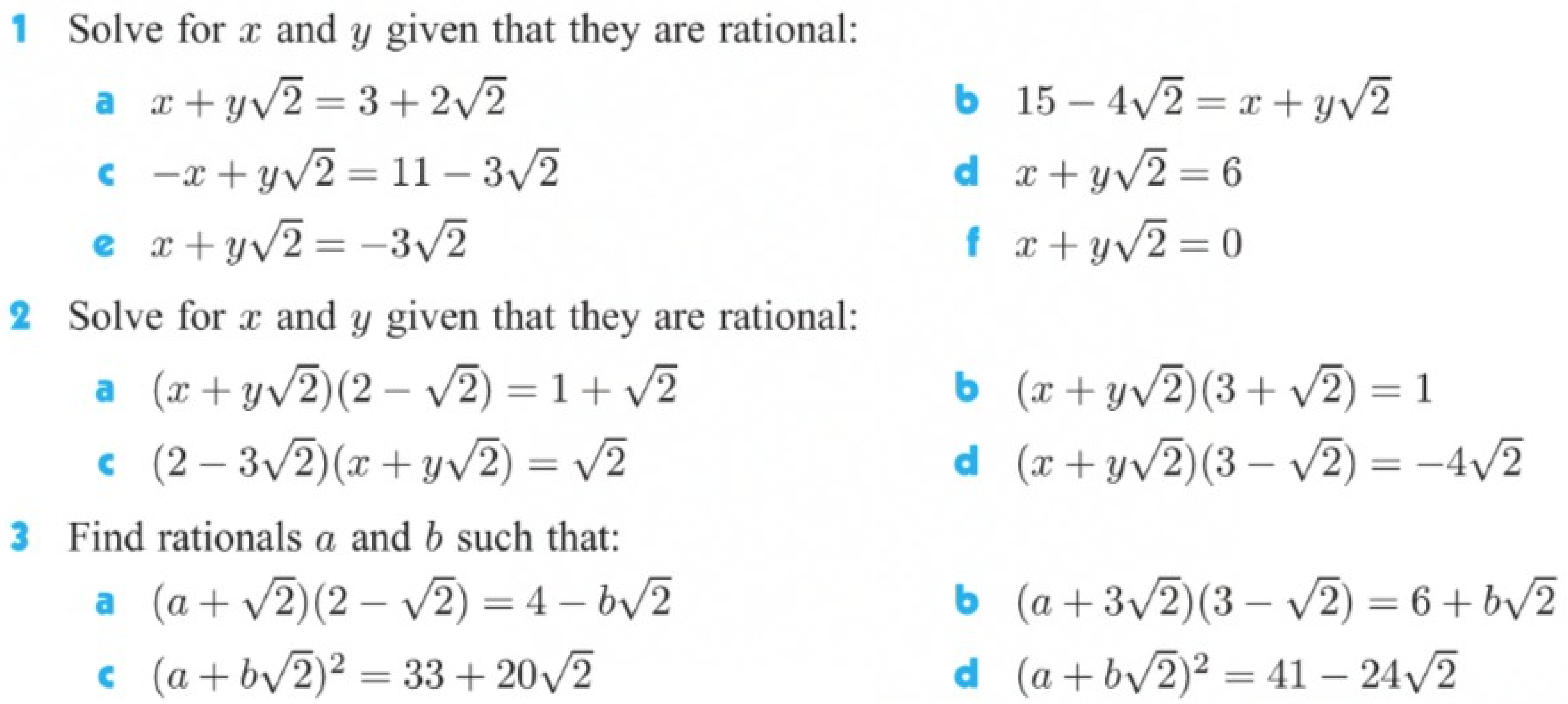


$$\begin{aligned}
(a+2\sqrt{2})(3-\sqrt{2}) &= 5+b\sqrt{2}\\
\therefore\ 3a-a\sqrt{2}+6\sqrt{2}-4 &= 5+b\sqrt{2}\\
\therefore\ (3a-4)+(6-a)\sqrt{2} &= 5+b\sqrt{2}
\end{aligned}$$

$\therefore$ since $\sqrt{2}$ is irrational, $3a-4=5$ and $6-a=b$

$\therefore\ 3a=9$

$\therefore\ a=3$ and hence $b=6-3=3$



**1** Solve for $x$ and $y$ given that they are rational:

**a** $x + y\sqrt{2} = 3 + 2\sqrt{2}$

**b** $15 - 4\sqrt{2} = x + y\sqrt{2}$

**c** $-x + y\sqrt{2} = 11 - 3\sqrt{2}$

**d** $x + y\sqrt{2} = 6$

**e** $x + y\sqrt{2} = -3\sqrt{2}$

**f** $x + y\sqrt{2} = 0$

**2** Solve for $x$ and $y$ given that they are rational:

**a** $(x + y\sqrt{2})(2 - \sqrt{2}) = 1 + \sqrt{2}$

**b** $(x + y\sqrt{2})(3 + \sqrt{2}) = 1$

**c** $(2 - 3\sqrt{2})(x + y\sqrt{2}) = \sqrt{2}$

**d** $(x + y\sqrt{2})(3 - \sqrt{2}) = -4\sqrt{2}$

**3** Find rationals $a$ and $b$ such that:

**a** $(a + \sqrt{2})(2 - \sqrt{2}) = 4 - b\sqrt{2}$

**b** $(a + 3\sqrt{2})(3 - \sqrt{2}) = 6 + b\sqrt{2}$

**c** $(a + b\sqrt{2})^2 = 33 + 20\sqrt{2}$

**d** $(a + b\sqrt{2})^2 = 41 - 24\sqrt{2}$

**1** **a** $x = 3,\ \ y = 2$ **b** $x = 15,\ \ y = -4$

**c** $x = -11,\ \ y = -3$ **d** $x = 6,\ \ y = 0$

**e** $x = 0,\ \ y = -3$ **f** $x = y = 0$

**2** **a** $x = 2,\ \ y = \frac{3}{2}$ **b** $x = \frac{3}{7},\ \ y = -\frac{1}{7}$

**c** $x = -\frac{3}{7},\ \ y = -\frac{1}{7}$ **d** $x = -\frac{8}{7},\ \ y = -\frac{12}{7}$

**3** **a** $a = 3,\ \ b = 1$ **b** $a = 4,\ \ b = 5$

**c** $a = 5,\ \ b = 2$ or $a = -5,\ \ b = -2$

**d** $a = 3,\ \ b = -4$ or $a = -3,\ \ b = 4$